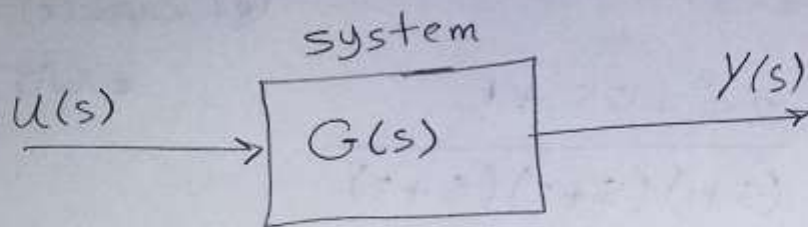


19 مراجعة نظم وانشاء

→ state-space model



$$\dot{x}(t) = A x(t) + B u(t)$$

$$y(t) = C x(t) + D u(t)$$

Given: $G(s) = \frac{Y(s)}{U(s)}$

(1) Controllable Canonical Form (CCF)

(2) observable " " (OCF)

(3) Paraller [diagonal] Form

Example $G(s) = \frac{Y(s)}{U(s)} = \frac{2s^2 + 5s + 1}{s^3 + 6s^2 + 11s + 6}$

different from each other

Sol

ch. eqn $\Rightarrow s^3 + 6s^2 + 11s + 6 \rightarrow \text{roots} = \text{Poles}$

(1) real and distinct

(2) real & equal.

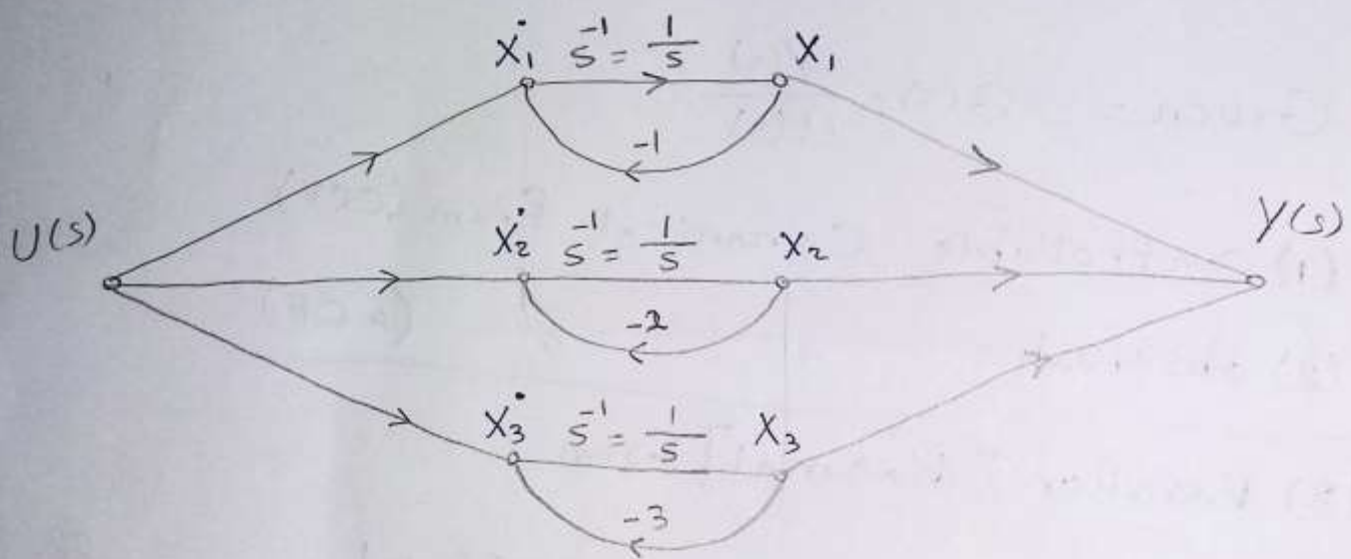
(3) Complex Conjugate.

Poles $\lambda_1 = -1$
 $\lambda_2 = -2$
 $\lambda_3 = -3$

من الممكن إيجاد في المسائل
معادلة تفاضلية ويطلب حلها بالطريقة
دي هينسب (Laplace) لها
والحل يكون كالعادة.

$$\therefore G(s) = \frac{2s^2 + 5s + 1}{(s+1)(s+2)(s+3)}$$

$$= \frac{K_1}{s+1} + \frac{K_2}{s+2} + \frac{K_3}{s+3}$$



diagonal form

Postmultiplication

من يجعل ال (branches)

الخارجية من $U(s)$ إلى

الداخلية من $K_3, K_2, K_1 = Y(s)$

$\cdot K_3$

Premultiplication

من يجعل ال (branches)

الخارجية من $K_3, K_2, K_1 = U(s)$

والداخلية من $Y(s) = (1, 1, 1)$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + D u(t)$$

$\rightarrow 0$

← تنتقل إلى (Signal Flow graph) وتعلم الأمانة في القانون
للسابق (القيم المضافة بالبرهان هي الحل)

4) Cascaded [series] ~~Form~~ Form

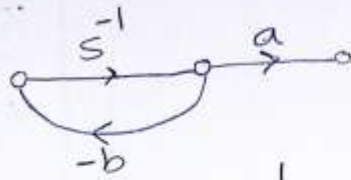
$$\text{Given: } G(s) = \frac{Y(s)}{U(s)} = \frac{10(s+3)(s+8)}{(s+4)(s+5)(s+6)}$$

$$\frac{Y(s)}{R(s)} = \frac{10s^2 + 110s + 240}{s^3 + 15s^2 + 74s + 120} \quad \leftarrow \text{وهو أساس المعادلة}$$

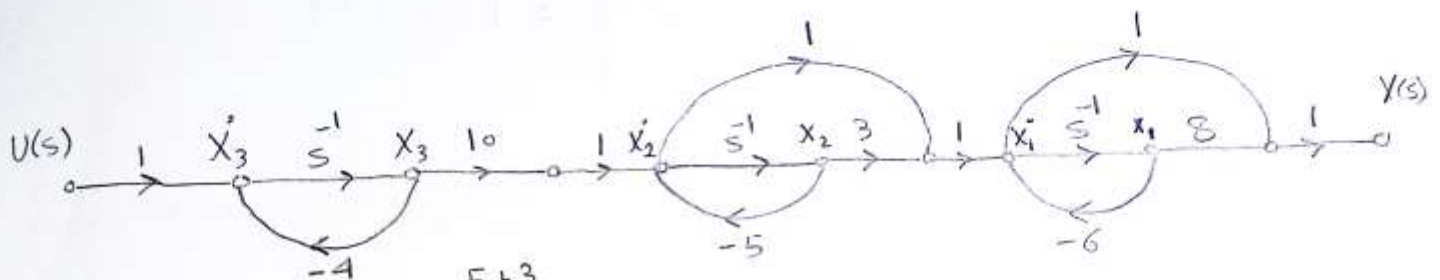
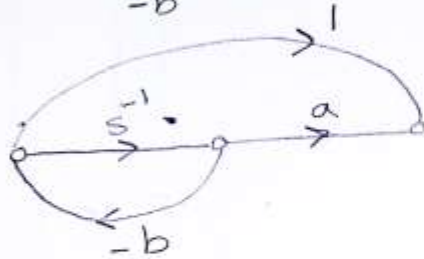
$$\frac{Y(s)}{U(s)} = \left(\frac{10}{s+4} \right) \cdot \left(\frac{s+3}{s+5} \right) \cdot \left(\frac{s+8}{s+6} \right)$$

Note

$$\frac{a}{s+b} \rightarrow$$



$$\frac{s+a}{s+b} \rightarrow$$



$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -6 & \underline{-2} & 10 \\ 0 & -5 & 10 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$\begin{matrix} \nearrow -5+3 \\ \nearrow 8+(-6) \end{matrix}$

$$y(t) = \begin{bmatrix} 2 & -2 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0 u(t)$$

$\begin{matrix} \nearrow 3+(-5) \end{matrix}$

← القيمة (-2) هي لها مصدرين 3 و -5 نجمعهم معاً

السؤال هييجي كده

[4] Lec 19

→ obtain two different models for state space representation.

→ time response of the system

Given: $G(s) = \frac{Y(s)}{U(s)} \Rightarrow Y(s) = G(s) \cdot U(s)$

$$= \left[G(s) \cdot \frac{1}{s} \right]$$

in unit step
 $u(t) = 1$

or

Given

$$\dot{x}(t) = A x(t) + B u(t)$$

$$y(t) = C x(t) + D u(t)$$

→ *
→ First-order D.E.

→ Find unit-step response

Note I $G(s) = \frac{Y(s)}{U(s)} = C (sI - A)^{-1} B + D$

(response) $Y(s)$ $G(s)$ هو القانون \Rightarrow ومنه نحسب

• الطريقة الثانية هي نعامل مع القانون *

$$x(t) = \phi(t) x(0) + \int_0^t \phi(t-\tau) B u(\tau) d\tau$$

where $u(\tau) = u(t) \Big|_{t=\tau}$

$$u(t) = 1 \Rightarrow u(\tau) = 1$$

Ex 1 P

$$X(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \\ \vdots \\ x_n(0) \end{bmatrix} = \text{initial state vector} = x(t) \Big|_{t=0}$$

$$\phi(t) = \begin{bmatrix} & \\ & \\ & \end{bmatrix}_{n \times n} = \text{state transition matrix}$$

$$\phi(t) = \mathcal{L}^{-1} (sI - A)^{-1}_{2 \times 2}$$

Given $\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$

$$y(t) = \begin{bmatrix} 1 & -1 \end{bmatrix} x(t)$$

Find: @ unit-step response for

$$x(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$y(t) = ??$ when
 $u(t) = 1 = \text{unit-step}$

Sol

$$\phi(t) = \mathcal{L}^{-1} (sI - A)^{-1}$$

$$sI - A = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$$

عبارة المخرج الرئيسي $\dot{x}(t)$
 فنحذفها في s

6/ Page 19

$$(sI - A)^{-1} = \begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix}$$

الحقار الموجود في المصفوفة هو قيم محددا للمصفوفة
 $(sI - A)$ $\neq s(s+3) - (-1 \times 2)$

$$\phi(t) = \mathcal{L}^{-1} \begin{bmatrix} \frac{2}{s+1} + \frac{-1}{s+2} & \frac{1}{s+1} + \frac{-1}{s+2} \\ \frac{-2}{s+1} + \frac{2}{s+2} & \frac{-1}{s+1} + \frac{2}{s+2} \end{bmatrix}$$

$$\phi(t) = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

$\phi(t) = \phi(t)$ \leftarrow لا زير (check) \leftarrow

$$\phi(0) = \phi(t) \Big|_{t=0} = I$$

$$\mathbb{I}_1 \rightarrow x(t) = \phi(t) \cdot x(0) + \underbrace{\int_0^t \phi(t-\tau) B u(\tau) \cdot d\tau}_{\mathbb{I}_1}$$

$$\dot{\mathbb{I}}_1 = \int_0^t \begin{bmatrix} 2e^{-(t-\tau)} & -e^{-2(t-\tau)} \\ -2e^{-(t-\tau)} & +2e^{-2(t-\tau)} \end{bmatrix} \begin{bmatrix} e^{-(t-\tau)} & -e^{-2(t-\tau)} \\ e^{-(t-\tau)} & +2e^{-2(t-\tau)} \end{bmatrix} d\tau$$

$$\cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} d\tau$$

$$= \int_0^t \begin{bmatrix} e^{-(t-\tau)} & -e^{-2(t-\tau)} \\ -e^{-(t-\tau)} & +2e^{-2(t-\tau)} \end{bmatrix} d\tau$$

$$= \begin{bmatrix} e^{-(t-\tau)} & -\frac{1}{2} e^{-2(t-\tau)} \\ -e^{-(t-\tau)} & +e^{-2(t-\tau)} \end{bmatrix} \bigg|_0^t = \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix} - \begin{bmatrix} e^{-t} & -\frac{1}{2} e^{-2t} \\ -e^{-t} & +e^{-2t} \end{bmatrix}$$

$$\mathbb{I}_1 = \begin{bmatrix} \frac{1}{2} - e^{-t} + \frac{1}{2} e^{-2t} \\ e^{-t} - e^{-2t} \end{bmatrix}_{2 \times 1}$$

Lec 19 [8]

$$X(t) = \underbrace{\begin{bmatrix} \\ \end{bmatrix}}_{\phi(t)} \underbrace{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}_{2 \times 1} + \underbrace{\begin{bmatrix} \frac{1}{2} e^{-t} + \frac{1}{2} e^{-2t} \\ e^{-t} - e^{-2t} \end{bmatrix}}_{2 \times 1}$$

$$X(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix} + \begin{bmatrix} \frac{1}{2} + \frac{1}{2} e^{-2t} \\ -e^{-2t} \end{bmatrix}$$

$$\therefore x_1(t) = \frac{1}{2} + \frac{1}{2} e^{-2t}$$

$$x_2(t) = -e^{-2t}$$

وضع $t=0$ حساب $x(0)$
 $\begin{bmatrix} 1 \\ -1 \end{bmatrix} = x(0)$

$$y(t) = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} + \frac{1}{2} e^{-2t} \\ -e^{-2t} \end{bmatrix}$$

$$y(t) = \frac{1}{2} + \frac{3}{2} e^{-2t} \rightarrow \text{unit-step response}$$

stability of system

\rightarrow stable
 \rightarrow unstable
 \rightarrow critically stable

Lec 9 [9]

→ check the system stability

لو ظهرت حتر يتم حلها بحسب Ch-ern للنظام .

حل على المثال
السابقة .

→ another solution:-

$$G(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$$

$A \rightarrow$ system matrix

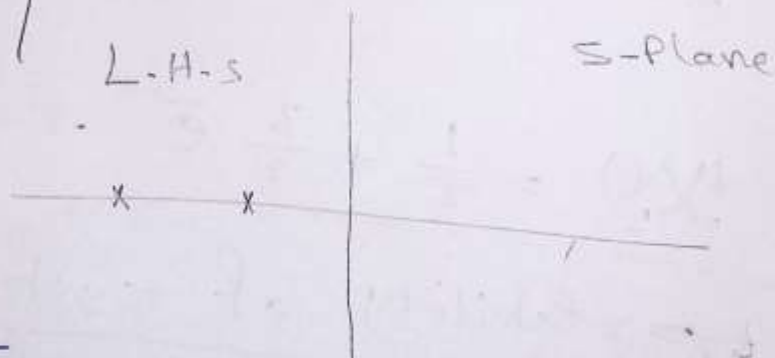
$$\text{Ch-ern} \Rightarrow |sI - A| = 0$$

$$|sI - A| = \begin{vmatrix} s & -1 \\ -2 & s+3 \end{vmatrix} = \boxed{s^2 + 3s + 2} = 0$$

Poles \equiv eigen values

$$\lambda_1 = -1, \lambda_2 = -2$$

نحن أن النظام
(stable)



Lec 19 10